Exam. Code : 206701
Subject Code : 4647

M.Sc. (Computer Science) Semester—I

DISCRETE STRUCTURES

Paper—MCS-104

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Attempt any **FIVE** questions.

1. (a) Consider the relation ~ defined on Z by declaring that a ~ b if and only if b + a is even number. Is ~ an equivalence relation ? If yes, prove that it is. If no, explain why not ?

(b) Is the function $y = f(x) = 3x + 2$, $x, y \in R$ onto ? Is it one-to-one ? What if $x, y \in Z$? Explain.

(c) Prove that $(A \cap B) \cup (A \cap \bar{B}) = A$, where $\bar{B}$ denotes the complement. 7,6,7

2. (a) Let R be a commutative ring with unity. If M is a maximal ideal of R, prove that R/M is a field.

(b) If R is a ring and f(x), g(x) in R[x] are degrees 3 and 4 respectively, then f(x)g(x) is always of degree 7. Is the above statement true or false ? Justify. 10,10

3. (a) Attempt each of the following :

   (i) Define a directed graph and a sub graph.

   (ii) True or False ? Justify. There is one and only one path between every pair of vertices in a tree.

   (b) Define edge connectivity and vertex connectivity. Give an example of a graph in which vertex connectivity is strictly smaller than edge connectivity. 10,10

4. (a) Show that the sum of the degrees of all vertices of a graph is twice the number of edges in the graph.

   (b) Define the edge chromatic number of a graph. Construct a graph with chromatic number 5.

   (c) If G is a planar graph, then every node in G has degree five or less. 7,6,7

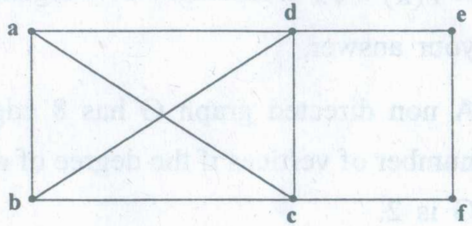
5. (a) Solve the recurrence relation :

   $a_n = 4a_{n-1} - 4a_{n-2}$; $n \geq 3$ $a_1 = 1$, $a_2 = 7$.

   (b) How many ways are there to distribute 40 identical jellybeans among 4 children without any restriction ?

(c) How many permutations are there of the 26 letters of the English alphabet that contain the sequence 'MATH' ? 10,5,5

6. (a) Define spanning tree and minimal spanning tree. Draw three spanning trees of the following graph :



(b) Let G be a graph having V vertices, E edges and K components, where each component is a tree. Obtain a formula in terms of V, E and K.

10,10

7. (a) Find a generating function to count the number of integer solutions to $e_1 + e_2 + e_3 = 10$ if for each i, $0 \leq e_i$.

(b) Explain the application of Boolean algebra in logic circuit and switching functions, by taking appropriate examples. 10,10

8. (a) State the principle of Inclusion/Exclusion for 3 sets. Use the principal of Inclusion and Exclusion to find integers between 100 and 10100, both inclusive, which are divisible by 2, 5 & 7.

(b) Determine the number of onto functions from {1, 2, ...,n} to {1, 2}, where n >= 2.

(c) Is $F(x) = \sqrt{x}$ a function ? Give argument to support your answer.

(d) A non directed graph G has 8 edges. Find the number of vertices if the degree of each vertex in G is 2.

(e) Prove that every field is an Integral Domain.

7,2,2,2,7